

# Mathematical Models of Systems

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## Dates and time

13-01-2025 (Stephan)

20-01-2025 (Stephan)

27-01-2025 (Felix)

03-02-2025 (Felix)

10-02-2025 (Felix)

17-02-2025 (Felix)

24-02-2025 (Stephan)

03-03-2025 (Stephan)

From 10.15-12.30 hrs

## Course location

Cursus- en vergadercentrum Domstad, Utrecht

## ECTS

6 ECTS if the homework is completed successfully

1.5 ECTS for auditing the course

## Lecturers

Assoc.-Prof. dr. F. Schwenninger, University of Twente

Assoc.-Prof. dr. S. Trenn, University of Groningen

## Objective

The purpose of this course is to discuss the ideas and principles behind modelling using the behavioral approach.

In the behavioral approach, dynamical models are specified in a different way than is customary in transfer function or state space models. The main difference is that it does not start with an input/output representation. Instead, models are simply viewed as relations among certain variables. The collection of all time trajectories which the dynamical model allows is called the behavior of the system. Specification of the behavior is the outcome of a modelling process. Models obtained from first principles are usually set-up by tearing and zooming. Thus the model will consist of the laws of the subsystems on the one hand, and the interconnection laws on the other. In such a situation it is natural to distinguish between two types of variables: the manifest variables which are the variables which the model aims at, and the latent variables which are auxiliary variables introduced in the modelling process. Behavioral models easily accommodate static relations in addition to the dynamic ones. A number of system representation questions occur in this framework, among others:

- the elimination of latent variables
- input/output structures
- state space representations

- differential-algebraic equations

Large parts of the course follow closely the book “Introduction to Mathematical Systems Theory: A Behavioral Approach” by J.W. Polderman and J.C. Willems (Springer 1998 as e-book) [PW98]

## Contents

1. General ideas. Mathematical models of systems. Dynamical systems. Examples from physics and economics. Linear time-invariant systems. Differential equations. Polynomial matrices. (Chapters 1+2 of [PW98])

2. *Discussion HW1 (peer grading)*. Minimal and full row rank representation. Autonomous systems. Inputs and outputs. Equivalence of representations. Differential systems with latent variables. State space models. I/S/O models. (Chapters 3+4 of [PW98])

3. Controllability and Observability (Chapter 5 of [PW98])

4. *Discussion HW 2 (peer grading)*. Elimination of latent variables (Chapter 6 of [PW98])

6. Elimination of latent variables in interconnected systems. From i/s/o to i/o and from i/o to i/s/o models. Canonical forms. (Chapter 6 of [PW98]).

7. *Discussion HW 3 (peer grading)*. Minimality. Image representation (Chapter 6 of [PW98]).

8.. Differential-algebraic equations. Regularity and Wong-sequences. Quasi-Weierstrass and quasi-Kronecker form

9. *Discussion HW 3 (peer grading)*. Controllability notions for DAEs.

## Prerequisites

The course is pretty much self-contained. Basic linear algebra and calculus should suffice.

## Course materials

The main reference is Introduction to Mathematical Systems Theory: A Behavioral Approach by J.W. Polderman and J.C. Willems (Springer 1998 as e-book).