

# Numerical Optimal Control

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## Dates and time

07-10-2024

14-10-2024

28-10-2024

04-11-2024

from 10:15-12:30

## Course location

Cursus- en vergadercentrum Domstad, Utrecht

## ECTS

3 ECTS if the homework is completed successfully

1 ECTS for auditing the entire course

## Lecturers

Dr. D. Krishnamoorthy, Eindhoven University of Technology

## Objective

The objective of this advanced-level course is to equip students with a comprehensive understanding of numerical methods for addressing optimal control problems prevalent in various engineering applications. Building upon the foundational knowledge acquired in Master's level courses on Model Predictive Control (MPC), which typically emphasize linear MPC formulations and their analysis, this course delves into the intricacies of nonlinear MPC (NMPC) and the numerical techniques essential for solving NMPC problems for dynamic systems.

Throughout the course, students will navigate the complexities of nonlinear dynamical systems, mastering the process of transcribing NMPC problems into finite horizon optimal control problems (FHOCP) solvable through nonlinear programming (NLP) methods. Specifically, students will explore Nonlinear Programming (NLP) within the context of optimal control, focusing on strategies to tackle non-convex NLPs utilizing Sequential Quadratic Programming (SQP) and Interior-Point methods.

Additionally, students will gain proficiency in various discretization techniques such as single shooting, multiple shooting, and collocation methods. By the course's conclusion, students will possess the skills to apply numerical techniques proficiently in the analysis and resolution of nonlinear optimal control problems. Moreover, they will acquire hands-on experience with the CasADi toolbox.

By bridging the gap between linear and nonlinear MPC, this course will enable students with the expertise necessary to tackle advanced MPC problems, thereby enhancing their capacity to engage in research in these areas.

## Contents

Lecture 1 – Understanding the KKT optimality conditions

- First and second order conditions of optimality
- Regularity conditions and constraint qualifications

- Sensitivity of NLPs – interpretation of Lagrange multipliers

#### Lecture 2 - Newton-type optimization algorithms

- Root finding with Newton-type methods
- Solving KKT conditions using Newton-type methods
- Sequential quadratic problems and active set solvers
- Interior point methods

#### Lecture 3 - Transcription of optimal control problems for ODE systems – Part 1

- Single shooting
- Multiple shooting

#### Lecture 4 - Transcription of optimal control problems for ODE systems – Part 2

- Direct Collocation
- NLP Parametric sensitivities

### Course materials

The lecture notes will be distributed during the course.

Additional recommended reading material: Chapter 8 of Rawlings, Mayne, Diehl 2017. Model Predictive Control.

### Prerequisites

The students are expected to be familiar with linear Algebra, Linear MPC, and fundamentals of optimization.

### Homework assignments

There are four homework assignments (one assignment per lecture) that will be distributed during the lectures. Each assignment must be handed in within two weeks. The exercises will be both pen-and-paper and computer based. The computer assignments will use [CasADi](#) (with MATLAB or Python). For the computer assignments students will have to submit a short report (in a template that will be provided) and software files in a zip archive. Homework is graded on a scale from 1 to 10. Missing assignments receive the grade 1. The final grade for the course is a weighted average of the grades for the homework sets.

### Teacher Profile:

Dinesh Krishnamoorthy is an Assistant professor at the Department of Mechanical Engineering at TU Eindhoven, where he is a part of the Control Systems Technology group. Dinesh currently teaches 4DM20 - Engineering Optimization (~150 Students) at TU Eindhoven, and received his UTQ in Nov 2023. He has also previously delivered a short course on Numerical Optimal Control at UFRJ in 2019. His research interests include distributed optimization, numerical optimal control, real-time optimization, and Bayesian optimization, with applications to energy systems.