Mathematical Models of Systems

Dates and time

15-01-2024
22-01-2024
29-01-2024
05-02-2024
from 10.15-12.30

12-02-2024
04-03-2024
11-03-2023
18-03-2023
25-03-2023 (attendance is required)
from 13.45-16.00

Course location

Cursus- en vergadercentrum Domstad, Utrecht

ECTS

6 ECTS if the homework is completed successfully
1.5 ECTS for auditing the entire course

Lecturers

Dr. J.W. Polderman, University of Twente
Prof. dr. S. Trenn, University of Groningen

Objective

The purpose of this course is to discuss the ideas and principles behind modelling using the behavioral approach, and to apply these ideas to control system design.

In the behavioral approach, dynamical models are specified in a different way than is customary in transfer function or state space models. The main difference is that it does not start with an input/output representation. Instead, models are simply viewed as relations among certain variables. The collection of all time trajectories which the dynamical model allows is called the behavior of the system. Specification of the behavior is the outcome of a modelling process. Models obtained from first principles are usually set-up by tearing and zooming. Thus the model will consist of the laws of the subsystems on the one hand, and the interconnection laws on the other. In such a situation it is natural to distinguish between two types of variables: the manifest variables which are the variables which the model aims at, and the latent variables which are auxiliary variables introduced in the modelling process. Behavioral models easily accommodate static relations in addition to the dynamic ones. A number of system representation questions occur in this framework, among others:

- the elimination of latent variables
- input/output structures
- state space representations

In the first part of the course, we will review the main representations, their interrelations, and their basic properties. In the context of control, we will view interconnection as the basic principle of design. In the to-be-controlled plant there are certain control terminals and the controller imposes additional laws on these terminal variables. Thus the controlled system has to obey the laws of both the plant and the controller. Control design procedures thus consist of algorithms that associate with a specification of the plant (for example, a kernel, an image, or a hybrid representation involving latent variables) a specification of the controller, thus passing directly from the plant model to the controller. We will extensively discuss the notion of implementability as a concept to characterize the limits of performance of a plant to be controlled.

Contents

1. General ideas. Mathematical models of systems. Dynamical systems. Examples from physics and economics. Linear time-invariant systems. Differential equations. Polynomial matrices.

2. Minimal and full row rank representation. Autonomous systems. Inputs and outputs. Equivalence of representations.

3. *Discussion HW1 (peer grading).* Differential systems with latent variables. State space models. I/S/O models.

4. Controllability. Controllable part. Observability.

5. *Discussion HW 2 (peer grading).* Elimination of latent variables

6. Elimination of latent variables in interconnected systems. From i/s/o to i/o.

7. *Discussion HW 3 (peer grading).* from i/o to i/s/o models. Canonical forms.

8. Minimality. Image representation. Full and partial interconnection.

9. *Discussion HW 3 (peer grading).* Control by interconnection.

Prerequisites

The course is pretty much self-contained. Basic linear algebra and calculus should suffice.

Course materials

The main reference is Introduction to Mathematical Systems Theory: A Behavioral Approach by J.W. Polderman and J.C. Willems (Springer 1998 as e-book).