

# Linear Matrix Inequalities in Control

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## Dates and time

13-11-2023

20-11-2023

27-11-2023

04-12-2023

from 13.45-16.00

## Course location

Cursus- en vergadercentrum Domstad, Utrecht

## ECTS

3 ECTS if the homework is completed successfully.

1 ECTS for auditing the entire course

## Lecturers

Prof.dr. S. Weiland, Eindhoven University of Technology

Dr. M.C.F. Donkers, Eindhoven University of Technology

## Objective

Linear matrix inequalities (LMIs) have proven to be a powerful tool to approach control problems that appear hard, if not impossible, to solve in an analytic fashion. The history of LMIs goes back to the forties and their role in control became emphasized in the sixties (Kalman, Yakubovich, Popov, Willems). Contemporary numerical interior-point methods and semi-definite programming techniques are increasingly powerful and allow solving LMIs in a practically efficient manner (Nesterov, Nemirovskii 1994). Several Matlab software packages are available that allow a simple coding of general LMI problems that arise in typical control problems.

Because of the availability of fast and efficient solvers for semi-definite programs, the research in robust control has experienced a paradigm shift towards reformulating control problems in terms of feasibility tests of systems of LMIs, where properties of convexity and semi-definite programs are fully exploited to solve relevant problems in systems and control.

The main emphasis of the course is:

- To reveal the basic principles of formulating desired properties of a control system using LMIs
- To demonstrate techniques that convert a controller synthesis problem into an LMI problem.
- To get familiar with the use of software packages for performance analysis and controller synthesis using LMI techniques.

The power of this approach is illustrated by several fundamental robustness and performance problems in analysis and design of linear control systems.

## Contents

1. Some facts from convex analysis. Linear Matrix Inequalities: Introduction. History. Algorithms for their solution.
2. The role of Lyapunov functions to ensure invariance, stability, performance, robust performance. Considered criteria: Dissipativity, integral quadratic constraints, H<sub>2</sub>-norm, H<sub>∞</sub>-norm, upper bound of peak-to-peak norm. LMI stability regions.
3. A general technique to proceed from LMI analysis to LMI synthesis. State feedback and output feedback synthesis algorithms for robust stability, nominal performance and robust performance using general scaling.
4. A choice of extensions to mixed control problems and to linear parameter-varying controller design, robust estimation problems or the use of multiplier techniques in control system design.

## Course materials

The main reference material for the course will be an extensive set of lecture notes by Carsten Scherer and Siep Weiland. Additional reference material:

1. S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, SIAM studies in Applied Mathematics, Philadelphia, 1994.
2. L. El Ghaoui and S.I.Niculescu (Editors), Advances in Linear Matrix Inequality Methods in Control, SIAM, Philadelphia, 2000.
3. A. Ben-Tal, A. Nemirovski, Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications, SIAM-MPS Series in Optimizatoin, SIAM, Philadelphia, 2001.
4. G. Balas, R. Chiang, et al. (2006). Robust Control Toolbox (Version 3.1), The MathWorks Inc.
5. J. Löfberg, YALMIP, <http://control.ee.ethz.ch/~joloef/yalmip.php>.

## Prerequisites

Linear algebra, calculus, basic system theory, MATLAB.

## Homework assignments

We plan to issue 4 homework sets that include choices of theoretical and practical assignments. Full credit is received for successfully solving the assigned take-home sets.