stability, relative stability and synchronization of dynamical systems with time-delay

lecturers

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objective

Time-delays are important components of many systems in engineering, economics and the life sciences, due to the fact that the transfer of material, energy and information is mostly not instantaneous. Time-delays appear for instance as computation and communication lags, they model transport phenomena and heredity and they arise as feedback delays in control loops. Since delays have a significant effect on the dynamic behavior of the system, and this effect is not always intuitive, it is important to take them explicitly into account in the mathematical model. Severe challenges in the research on time-delay systems are due to the emergence of new application fields, mainly in the area of large-scale interconnected systems and networks (e.g., analysis of neuronal networks, control of communication networks like the internet, networked control systems, distributed decision making and control).

The aim of the course is to present a detailed description of the main properties of dynamical systems subjected to time-delays, thereby highlighting differences and similarities with delay-free systems and providing insight, as well as to present an overview of techniques for the stability analysis of equilibria. Both frequency domain and time-domain methods are discussed. In the second part of the course, the emphasis is on the analysis of relative stability problems, such as consensus in multi-agent systems and synchronization. Several applications, ranging from traffic flow analysis to networks of interacting neurons, complete the presentation.

contents

1: Introduction to time-delay systems
- Examples and applications, fundamental properties of time-delay systems (representations, existence and uniqueness of solutions, stability notions), spectral properties of linear time-delay systems
- Qualitative effects of delays in dynamical systems (instability mechanisms, limitations in control, opportunities to use delays in controllers).

2: Methods for stability analysis

- Frequency domain techniques : spectral analysis (characteristic roots and pseudospectral abscissa computation), stability regions in parameter spaces (from D-subdivision to numerical continuation, invariance properties and crossing direction of roots, algebraic and geometric techniques)
- Time-domain methods: generalization of Lyapunov's second method (Lyapunov-Krasovskii functionals, Lyapunov-Razumikhin functions) , methods based on the small gain theorem.

3: Relative stability problems

- Notions of relative stability; consensus and synchronization problems
- Synchronization of two coupled systems: synchronization manifold, stabilization mechanism, sufficient conditions, effect of coupling delays (invasive versus non-invasive coupling, synchronization conditions in terms of coupling gain and delay parameters, delay compensation)

4: Synchronization in complex networks

- Description of network, including the analysis of its graph Laplacian; relation between synchronization and the network topology, full and partial synchronization
- Examples: networks of coupled Hindmarsh-Rose neurons

prerequisites

Basic courses on dynamical systems and control

homework assignments

Two homework assignments will be given (at the end of Lectures 2 and 4), where the students are asked to solve exercises with pen and paper and MATLAB. Software, its documentation and supplementary course material will be made available online.