

# Underactuated mechanical systems: motion planning, motion representation and motion control methods

## lecturers

Prof. A.S. Shiriaev, NTNU, Norway. Email: anton.shiriaev@ntnu.no

Dr. A.Y. Pogromskiy, Eindhoven University of Technology. Email: a.pogromski@tue.nl

## objective

The course helps students systematically explore several topics and research directions of modern robotics and nonlinear control theory linked to efforts in developing scalable methods for performing and analyzing agile movements of dynamically constrained robotic systems. Demands for automating various labor-intensive tasks such as grasping, manipulating or handling of external objects (performed nowadays in industry and service applications primarily by humans) motivate for model-based motion planning and control methods for such systems. Similarly, dynamic and actuation constraints are dominating in developing aggressive maneuvers for flying machines; they are important for searching and controlling gaits of walking robots etc.

Most of dynamic constraints in applications are case specific or linked to scenarios of work of mechanisms. Meanwhile, some constraints are generic and can be simultaneously present in describing behaviors of quite distant nonlinear systems. Constraints due to underactuation provide examples of such generic structural features of nonlinear mechanical systems. They appear due to system designs and literally mean an excess of a number of degrees of freedom of the system (that are representative for a searched movement) over a number of actuators available in the system (for performing the movement). For a nonlinear mechanical system, the Newton second law describing the dynamics of non-actuated variables can be interpreted as a dynamic constraint defining a continuum set of equalities, which a movement of the system must obey and which are parameterized by a time interval the motion is defined. Such non-integrable relations can be irrelevant for some tasks. For instance, underactuation constraints in local regulating a position of a drone might have a negligible effect. In similar fashion, the constraint can be ignored in process of planning and controlling of some of slow motions of a flying machine. Meanwhile, the constraints are dominant and should be taken into account in planning and performing its agile maneuvers.

The lectures provide introductory materials and problem settings to the subject exploiting generic arguments for modelling, representing and analyzing behaviors of nonlinear systems, which can be further extended and applied for controlled mechanical systems. The development is well illustrated by solving a number of comprehensive examples of increased complexity, where important mathematical concepts and tools are emphasized and exemplified. The common thread of the lectures is put on discussion of formalization of motion planning, stabilization and stability analysis assignments and on discussion of integrated approaches for solving the tasks, which are relevant to engineering applications and practice. To the end, one of lectures is devoted to the analysis and control system design for the robotic system developed for performing non-prehensile manipulation tasks.

## contents

Lecture 1: Concepts of stability of a motion. Analytical and computational tools for detecting and for analysis of cycles of nonlinear dynamic systems (Lyapunov lemma, Poincare first return map, small parameter and Krylov-Bogolyubov methods for approximate integration, Andronov theorem). Examples.

Lecture 2: Nonlinear mechanical systems with constraints. Classification of constraints. Stability of nonlinear mechanical systems with constraints. Problem formulations and settings for motion (trajectory) planning for constrained controlled mechanical systems. Concepts of a motion generator (MG) and its dynamics for mechanical systems. A nested representation of motion candidates for underactuated mechanical systems. Properties of the dynamics of a MG derived

based on the nested representation of a behavior of an underactuated mechanical system. Examples of choices of MG and steps in planning feasible behaviors

Lecture 3: Concepts of transverse dynamics and moving Poincare sections for a motion of mechanical system. Analytic choices of transverse coordinates and their linearization for a motion of mechanical system. Tools for controlling a motion of mechanical system based on transverse linearization. Hybrid transverse linearization for analysis of stability and for stabilization of movements of hybrid mechanical systems (walking and running machines). Examples.

Lecture 4: The case study: non-prehensile manipulation of a passive disc rolling on a curved hand of the Butterfly robot. Choices for coordinates appropriate for representing unilateral and non-slipping constraints. Dynamics of the Butterfly robot in alternative sets of excessive coordinates. Steps in planning perpetual rotations of a passive disc on the hand of the Butterfly robot. Choices for a motion generator and for parametric sets of synchronization functions in searching feasible rotations. Transverse dynamics and transverse coordinates in a vicinity of the nominal rolling of a passive disc on the hand of the Butterfly robot. Transverse linearization and its robust stabilization. Experimental verification. Adaptivity and learning in performing non-prehensile manipulations.

### ***course materials***

The lecture notes and slides will be distributed during the course.

### ***prerequisites***

Basic background in mechanics, systems and control theory. Basic programming skills in Matlab.

### ***homework assignments***

Two homework assignments will be handed out at the end of the second and forth lectures. Each assignment will contribute up to 50% of the final grade for this course.