

# Computational Linear Algebra: A least squares perspective

## lecturers

Prof. M. Verhaegen, Delft University of Technology  
Prof. B. De Moor, KU Leuven

## objective

The computational aspects in solving identification and control problems is becoming more important with the event of embedded, cheap and general purpose control systems. Inspired by the evolution in hardware we observe an increase in the size of system theoretical problems that are analyzed in industry.

The numerical aspects of efficiency and accuracy in the designing numerical solutions is the driving force to computational linear algebra. The objective of this course is to show the use of linear algebra, its geometric interpretation and convex optimization in deriving new, simpler and easier to understand solutions to various system theoretical problems. In this course a number of key problems in system theory are formulated, solved and analyzed from a computational algebra perspective

## contents

### (a) Subspace identification (SI) of LTI systems

Basic linear algebra tools such as the QR factorization, the SVD and linear least squares problems allow to approximate structural information about LTI systems, such as the observability matrix, from input-output measurements. The latter data is assumed to be taken from an LTI system operating in open-loop. Different perturbation scenarios of the data are considered.

### (b) Nuclear Norm convex optimization for identifying LTI systems

Subspace identification methods are characterized o.a. by the calculation of low rank approximation in order to retrieve key subspaces of interest. Generally the SVD is used for that purpose. New families of subspace identification methods have emerged that replace the SVD step by nuclear norm optimization. A brief introduction will be given to this new development.

### (c) Realization theory and distance measures between linear systems.

It turns out that there are important and surprising connections between system theory (stochastic realization, subspace principle angles, cepstral norm and distances), information theory (Shannon entropy and mutual information), statistics (canonical correlations) and signal processing (Kalman filter).

We will elaborate in detail on these connections, develop the relation with subspace identification algorithms and show how these insights can be used in datamining, more specifically in the clustering of time series.

### (d) Applications of realization theory, subspace identification and distance measures.

It is little known that there are numerous applications of realization theory and subspace identification. We will elaborate on the roots of systems of multivariable polynomials in numerical algebraic geometry, eigenfrequencies and modes in modal analysis in mechanical engineering, subspace modelling from power spectra, the so-called shape from moments problem in computer tomography and the modelling of textures in one and more dimensions. Several industrial examples of subspace identification will be discussed.

## prerequisites

A master's degree in engineering with specialisation in signal, systems and/or control.

## lecture notes

Copies of all slides, and material contained in

- Book: M. Verhaegen and V. Verdult, "Filtering and System Identification: A least squares Approach", Cambridge University Press 2007.
- Book: Van Overschee P., De Moor B., Subspace Identification for Linear Systems: Theory,

Implementation, Applications, Kluwer Academic Publishers, 1996, 254 p., Downloadable from <http://homes.esat.kuleuven.be/sistawww/cgi-bin/pub.pl>

- PhD Thesis: De Cock K., Principal Angles in System Theory, Information Theory and Signal Processing, PhD thesis, Faculty of Engineering, KU Leuven (Leuven, Belgium), May 2002, 337 p. Downloadable from <http://homes.esat.kuleuven.be/sistawww/cgi-bin/pub.pl>.

### ***homework assignments***

The grading is based on 2 take-home exams that will be distributed to the students during the course