

# Multi-agent Network Dynamics and Games

## **lecturers**

Prof. dr. ir. M. Cao, University of Groningen  
Dr. ing. S. Grammatico, Eindhoven University of Technology

## **objective**

The aim of the course is to introduce the mathematical tools for analyzing the dynamics of autonomous, rational agents that interact over and evolve on networks. Application examples will be drawn from several domains, such as power systems, smart grids, network congestion control, social networks, robotic and sensor networks.

The selected mathematical tools are within linear algebra, graph theory, fixed point and monotone operator theory, computational and evolutionary game theory.

## **contents**

- Lecture 1: Introduction to network dynamics and games.  
Elements of graph theory and game theory: directed and undirected graphs. Multi-player, aggregative and network, games. Application to decision-making dynamics in social networks.
- Lecture 2: Multi-agent systems: an operator theoretic perspective.  
Elements of operator theory: fixed points, zeros, contraction, averaged and nonexpansive mappings. Fixed point algorithms: Banach iteration, Krasnoselskij iteration. Zero-finding algorithms: operator splitting, forward–backward algorithm. Application to network congestion control.
- Lecture 3: Introduction to evolutionary games.  
Evolutionary stable strategies (ESS), social optimality, discrete-time vs. continuous-time models. Application to the modeling of emergence of cooperation in self-organized systems.
- Lecture 4: Propagation dynamics in multi-agent networks.  
Linear threshold models, Markov chains, fast-slow dynamics. Applications to epidemic dynamics and behavioral propagation on complex networks.

## **course materials**

The lecture slides and notes will be made available on the DISC course platform.

The main textbook references are the following.

- [1] H. H. Bauschke and P. L. Combettes. Convex analysis and monotone operator theory in Hilbert spaces. Springer, 2010.
- [2] N. Biggs. Algebraic Graph Theory. Cambridge University Press, 2nd edition, 1994.
- [4] C. D. Godsil and G. F. Royle. Algebraic Graph Theory. Springer, 2001.
- [5] R.T. Rockafellar and R.J.B. Wets. Variational Analysis. Springer, 1998.
- [6] W. H. Sandholm. Population Games and Evolutionary Dynamics. MIT Press, 2010..

## **prerequisites**

The students are expected to be familiar with ordinary differential equations, linear control systems and linear algebra.

## **homework assignments**

Homework assignments will be handed out and graded. Some homework assignments may be replaced by an elective simulation project on multi-agent network games.