Linear Matrix Inequalities in Control

lecturers

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objective

Linear matrix inequalities (LMI's) have proven to be a powerful tool to approach control problems that appear hard if not impossible to solve in an analytic fashion. Although the history of LMI's goes back to the forties with a major emphasis of their role in control in the sixties (Kalman, Yakubovich, Popov, Willems), the present numerical interior point and semi-definite programming techniques are increasingly powerful to solve LMI's in a practically efficient manner (Nesterov, Nemirovskii 1994). Several Matlab software packages are available that allow a simple coding of general LMI problems that arise in typical control problems.
Because of the availability of fast and efficient solvers for semi-definite programs, the research in robust control has experienced a paradigm shift towards reformulating control problems in terms of feasibility tests of systems of LMI's where properties of convexity and semi definite programs are fully exploited to solve relevant problems in systems and control.

The main emphasis of the course is:

- to reveal the basic principles of formulating desired properties of a control system in the form of LMI's
- to demonstrate the techniques how to reduce the corresponding controller synthesis problem to an LMI problem.
- to get familiar with the use of software packages for performance analysis and controller synthesis using LMI tools.

The power of this approach is illustrated by several fundamental robustness and performance problems in analysis and design of linear control systems.

contents

1. Some facts from convex analysis. Linear Matrix Inequalities: Introduction. History. Algorithms for their solution.
2. The role of Lyapunov functions to ensure invariance, stability, performance, robust performance. Considered criteria: Dissipativity, integral quadratic constraints, H2-norm, H8-norm, upper bound of peak-to-peak norm. LMI stability regions.
3. Frequency domain techniques for the robustness analysis of a control system. Integral Quadratic Constraints. Multipliers. Relations to classical tests and to µ-theory.
4. A general technique to proceed from LMI analysis to LMI synthesis. State-feedback and outputfeedback synthesis algorithms for robust stability, nominal performance and robust performance using general scaling.
5. A choice of extensions to mixed control problems and to linear parametrically-varying controller design, robust estimation problems or the use of multiplier techniques in control system design.

course materials

The main reference material for the course will be an extensive set of lecture notes by Carsten Scherer and Siep Weiland. Additional reference material:

1. S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, SIAM studies in Applied Mathematics, Philadelphia, 1994.
2. L. El Ghaoui and S.I.Niculescu (Editors), Advances in Linear Matrix Inequality Methods in Control, SIAM, Philadelphia, 2000.
3. A. Ben-Tal, A. Nemirovski, Lectures on Modern Convex Optimization: Analysis, Algorithms, and

Engineering Applications, SIAM-MPS Series in Optimizaton, SIAM, Philadelphia, 2001.
4. G. Balas, R. Chiang, et al. (2006). Robust Control Toobox (Version 3.1), The MathWorks Inc.
5. J. Löfberg, YALMIP, http://control.ee.ethz.ch/˜joloef/yalmip.php.

prerequisites

Linear algebra, calculus, basic system theory, MATLAB.

homework assignments

We plan to issue 4 homework sets that include choices of theoretical and practical assignments. Full credit is received for successfully solving the assigned take-home sets.