Multi-Agent Network Dynamics and Games

lecturers

Dr. Sergio Grammatico, Delft University of Technology
Dr. Giulia Giordano, Delft University of Technology

objectives

The aim of the course is to introduce the mathematical tools for analyzing the dynamics of autonomous, rational agents that interact over and evolve on networks.
Application examples will be drawn from several domains, such as biological systems, power systems, smart grids, network congestion control, social networks, robotic and sensor networks.
The selected mathematical tools are within linear algebra, graph theory, fixed point and monotone operator theory.

contents

Introduction to network systems - Networks as graphs
Elements of graph theory: directed and undirected graphs; node degrees; weighted, balanced and regular graphs; node centrality measures; paths, walks and connectivity. Elements of algebraic graph theory: matrices defined over graphs, adjacency matrix, Laplacian matrix, incidence matrix.

Dynamics over networks
Linear dynamics defined over networks: flow dynamics and distributed averaging. Positive and compartmental systems, flow optimisation over networks, flow control. Consensus and multi-agent coordination problems.

Multi-agent systems - An operator-theoretic perspective
Elements of operator theory: fixed points, zeros, contraction, averaged and nonexpansive mappings. Fixed point algorithms: Banach iteration, Krasnoselskij iteration. Zero finding algorithms: splitting, forward–backward algorithm.

Multi-agent games
Nash equilibrium problem, best response mapping, decentralized and distributed equilibrium seeking algorithms. Generalized Nash equilibrium problem, preconditioned forward–backward equilibrium seeking, forward–backward–forward equilibrium seeking. Application to network congestion control.

course material

Lecture and notes; selected topics in Bullo, “Lectures on network systems”, and Bauschke, and Combettes, “Convex analysis and monotone operator theory in Hilbert spaces. A list of references can be found on the DISC course platform.

prerequisites

The prerequisite knowledge consists of linear algebra, calculus and optimization at a graduate level.

homework assignments

Grading by two take-home exams