

# Input Design and Parameter Estimation for Non-linear Systems

## lecturers

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## objective

Optimal input design is a classical problem in the identification literature. One way to tackle this problem is through optimal parametric sensitivity control, using Pontryagin's minimum principle. Unlike the traditional open-loop input design methods, in this course the emphasis is on finding a control law that maximizes parameter sensitivities for a *specific* set of parameters in a general single input-single state-single output nonlinear state-space model that is affine in the input.

After designing an optimal input and feeding this to the system subsequently experimental data can be collected and parameters can be estimated, given the input and output data and a prior model structure. Alternatively, a combination of recursive parameter estimation and optimal input design is also demonstrated as an elegant strategy for successful identification of a nonlinear system model.

The objective of the course is to present a methodology for (i) the design of optimal input signals for minimum-variance estimation of model parameters in the class of low-dimensional, nonlinear, state-space models under indirect state measurements, (ii) the estimation of parameters in a dynamic, non-linear model, and (iii) the introduction of the necessary concepts to apply an *adaptive* optimal input design strategy to nonlinear state space models.

At the end of the course the student should be able to:

- Understand the concepts of parameter sensitivity and optimal parametric sensitivity control;
- Compute parameter sensitivity trajectories together with model output trajectories;
- Design optimal input signals for low-order non-linear systems, simulate the corresponding response and estimate the unknown parameter(s);
- Evaluate the input design and parameter estimation results;
- Use off- and online methods for parameter estimation and model structure identification.

## contents

Lecture 1 - Introduction and background

- Introduction to course
- Parameter sensitivity and the Fisher Information Matrix
- Linear regression and Least squares estimation
- Static gain observers

Lecture 2 – Optimal parametric sensitivity control

- Pontryagin's minimum principle
- Singular optimal control (SOC)
- SOC for optimal input design (OID)
- Examples

Lecture 3 - Adaptive optimal parametric sensitivity control

- Framework
- Recursive parameter estimation
- Model Predictive Control
- Adaptive OID

Lecture 4 – Identification of non-linear systems

- Introduction to non-linear parameter estimation
- Role of parameter estimation in system identification
- Real-world example

### ***course materials***

1. Keesman, KJ, System Identification: an Introduction. Springer Verlag, UK, 2011
2. Keesman KJ and Walter E, Optimal input design for model discrimination using Pontryagin's maximum principle: Application to kinetic model structures. Automatica 50(5) : 535 – 1538, 2014
3. Stigter JD and Keesman KJ, Optimal parametric sensitivity control of a fed-batch reactor Automatica 40 (8) : 1459 – 1464, 2004.
4. Stigter JD, Vries D and Keesman KJ, On adaptive optimal input design: A bioreactor case study. AIChE Journal 52 (9) : 3290 – 3296, 2006.

Additional material distributed during the course.

### ***prerequisites***

Notions of optimal control theory and system identification at the intermediate level. Notions of Pontryagin's minimum principle and statistical estimation uncertainty might turn out useful as well. Extra course material on these topics will be provided to take into account different entry levels. Intermediate MATLAB programming skills are also required.

### ***homework assignments***

Two homework assignments, each one accounting for 50% of the final grade. The homework assignments, distributed at the end of the second and the fourth lecture, will consist of both mathematical and programming problem solving exercises.