mathematical models of systems

lecturers

Dr. J.W. Polderman, University of Twente
Prof. dr. K. Camlibel, University of Groningen

objective

The purpose of this course is to discuss the ideas and principles behind modelling
using the behavioral approach, and to apply these ideas to control system design.
In the behavioral approach, dynamical models are specified in a different way than is customary in transfer function or state space models. The main difference is that it does not start with an input/output representation. Instead, models are simply viewed as relations among certain variables. The collection of all time trajectories which the dynamical model allows is called the behavior of the system. Specifcation of the behavior is the outcome of a modelling process. Models obtained from first principles are usually set-up by tearing and zooming. Thus the model will consist of the laws of the subsystems on the one hand, and the interconnection laws on the other. In such a situation it is natural to distinguish between two types of variables: the manifest variables which are the variables which the model aims at, and the latent variables which are aux-iliary variables introduced in the modelling process. Behavioral models easily accommodate static relations in addition to the dynamic ones. A number of system representation questions occur in this framework, among others:

- the elimination of latent variables
- input/output structures
- state space representations

We will also introduce some important system properties as controllability and observability in this setting.
In the first part of the course, we will review the main representations, their interrelations, and their basic properties. In the context of control, we will view interconnection as the basic principle of design. In the to-be-controlled plant there are certain control terminals and the controller imposes additional laws on these terminal variables. Thus the controlled system has to obey the laws of both the plant and the controller. Control design procedures thus consist of algorithms which associate with a specifcation of the plant (for example, a kernel, an image, or a hybrid representation involving latent variables) a specifcation of the controller, thus passing directly from the plant model to the controller. We will extensively discuss the notion of implementability as a concept to characterize the limits of performance of a plant to be controlled. We will discuss how the problems of pole-placement and stabilization look like in this setting.

contents

1. General ideas. Mathematical models of systems. Dynamical systems. Examples from physics and economics. Linear time-invariant systems. Differential equations. Polynomial matrices.
2. Minimal and full row rank representation. Autonomous systems. Inputs and outputs. Equivalence of representations.

3. Differential systems with latent variables. State space models. I/S/O models.
4. Controllability. Controllable part. Observability.
5. Elimination of latent variables. Elimination of state variables.
6. From I/O to I/S/O models. Image representations.
7. Interconnection. Control in a behavioral setting. Implementability
8. Stability. Stabilization and pole placement.

course material

The main reference is Introduction to Mathematical Systems Theory: A Behavioral Approach by J.W. Polderman and J.C. Willems (Springer 1998 as e-book).

prerequisites

The course is pretty much self-contained. Basic linear algebra and caculus should suffice.